Closed-Box Loudspeaker Systems
Part II: Synthesis

RICHARD H. SMALL

School of Electrical Engineering, The University of Sydney
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Part I of this paper provides a basic low-frequency analysis of the closed-box loudspeaker system with emphasis on small-signal and large-signal behavior, basic performance limitations, and the determination of important system parameters from voice-coil impedance measurements. Part II discusses some important implications of the findings of Part I and introduces the subject of system synthesis: the complete design of loudspeaker systems to meet specific performance goals. Given a set of physically-realizable system performance specifications, the analytical results of Part I enable the system designer to calculate directly the required specifications of the system components.

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8. DISCUSSION

Driver Size

It has long been an accepted principle that a large bass driver is better than a small one. While this attitude seems to be justified by experience, it has recently been called into question [22]. The analysis in this paper demonstrates that driver size alone does not determine or limit system performance in areas of small-signal response, efficiency, or displacement-limited power capacity.

A large driver inevitably costs more than a small driver having identical small-signal and large-signal parameters of the kind discussed here. However, it is physically easier to obtain a large value of V_D and hence a high acoustic power capacity from a large driver, and the modulation distortion [23] produced by a large driver will be less than that of a small driver delivering the same acoustic output power.

Thus a large driver has no inherent advantage over a small one so far as small-signal response and efficiency are concerned. It may in fact have a cost disadvantage. But where high acoustic output at low distortion is required, the large driver has a definite advantage.

Enclosure Size

It is clear from section 4 that an air-suspension system having a high compliance ratio can duplicate the performance of a larger conventional closed-box system having a low compliance ratio. However, once the compliance ratio is made larger than about 4, there is no way to gain a significant reduction in enclosure size without affecting system performance.

A small air-suspension system, when compared to a large air-suspension system, must have a higher cutoff frequency, or lower efficiency, or both. As has been claimed many times, it is possible to design a small system to have the same *response* as a large system. But if both are non-wasteful air-suspension designs, then as shown by (26) or Fig. 8 the efficiency of the small system must be lower than that of the large system in direct proportion to size.

It is often possible to provide the same maximum acoustic output as well as the same response from the small system, but the lower efficiency of this system will dictate a higher input power rating and therefore a driver voice coil capable of dissipating more heat. Also, it is easily shown that for these conditions the driver of the small system will require a larger magnet (e.g., a heavier diaphragm of the same size may be driven through the same displacement, or a smaller diaphragm of the same mass may be driven through a larger displacement). Thus for this condition the driver for the small system must be more expensive than that for the large system.

It may be concluded that the pressure to design more and more compact high-quality loudspeaker systems leads directly to systems of reduced efficiency and, in most cases, reduced acoustic power capacity. If acoustic power capacity is not sacrificed, these compact systems require expensive drivers and must be used with powerful amplifiers.

Performance Specifications

Of all the components used in audio recording and reproduction, loudspeaker systems have the least complete and least informative performance specifications. In the low-frequency range at least, this need not be so.

If a specified voltage is applied to a direct-radiator loudspeaker system, the output of the system at low frequencies may be expressed in terms of an acoustic volume velocity which is *substantially independent of the acoustic load* [12], [24]. The "response" of a loudspeaker system expressed in this way is meaningless to most loudspeaker users, but a specification of the acoustic power or distant sound pressure delivered into a standard free-field load by this volume velocity is both meaningful and useful.

While the sound pressure delivered to a room is different from that delivered to a free field, the difference clearly is a property of the room, not of the loudspeaker system. If the room performance is very poor, it can be corrected acoustically or, in some cases, equalized electronically. This is in no way a deterrent to accurate specification of the basic loudspeaker system response by using a standard free-field load. In fact, the findings of Allison and Berkovitz [25] indicate that a 2π sr free-field load is a very reasonable approximation to a typical room load.

Such a standard-load approach has of course been used for years in loudspeaker measurement standards [18], [26], [27]. If it were applied more universally, it would provide a very useful—and presently unavailable—quantitative means of comparing loudspeaker systems. It is a particularly attractive method for specifying the low-frequency response of a system, because the nominal free-field low-frequency response and reference efficiency can be obtained quite easily from the basic parameters of the system.

A few manufacturers already supply these basic parameters or the directly-related free-field response and efficiency data. The practice deserves encouragement.

Typical System Performance

A sampling of closed-box systems of British, American and European origin was tested in late 1969 by measuring the system small-signal parameters as described in section 6. The frequency response and efficiency were then obtained from the relationships of sections 3 and 4.

Resonance frequencies (f_C) varied from 40 Hz to 90 Hz. Total Q (Q_{TCO}) varied from 0.4 to 2.0. Reference efficiencies (η_o) varied from 0.28% to 1.0%. While there was no general pattern of parameter combinations, all but a few of the systems fell into one of two categories:

1) Cutoff frequency (f_3) below 50 Hz with little or no peaking (Q_{TCO} up to 1.1). Size generally larger than 40 dm³ (1.4 ft³).
2) Cutoff frequency above 50 Hz with definite peaking (Q_{TCO} between 1.4 and 2.0). Size smaller than 60 dm³ (2 ft³)

One explanation for this situation was spontaneously provided (and demonstrated) by a salesman who sold American systems in both categories. Only category 1 systems would reproduce low organ and orchestral fundamentals, while category 2 systems had demonstrably stronger bass on popular music. Sales thus tended to be determined by the musical tastes of the customer. There is marketing sense in this, and economic sense as well, because the same driver which has category 1 performance in a large enclosure has category 2 performance—with a higher acoustic power capacity—in a small enclosure.

9. SYSTEM SYNTHESIS

System-Driver Relationships

The majority of closed-box systems operate with amplifiers having negligible output resistance, have a total moving mass no greater than that of the driver on a baffle, and obtain most of their total damping from electromagnetic coupling and mechanical losses in the driver. For these conditions, (7), (9), (13), (17) and (18) may be used to derive

$$\frac{Q_{TCO}}{Q_{TS}} \approx \frac{Q_{EC}}{Q_{ES}} = \frac{f_C}{f_S} = (\alpha + 1)^{1/2}, \qquad (54)$$

and thus

$$f_C/Q_{TCO} \approx f_S/Q_{TS}, \qquad (55)$$

where Q_{TS} is the total Q of the driver at f_S for zero source resistance [12, eq. (47)], i.e.,

$$Q_{TS} = Q_{ES}Q_{MS}/(Q_{ES} + Q_{MS}). \qquad (56)$$

These equations show that for any enclosure-driver combination (i.e., value of α) the system resonance frequency and Q will be in the same ratio as those of the driver, but individually raised by a factor $(\alpha + 1)^{1/2}$. This increase is plotted as a function of α in Fig. 13.

This approximate relationship and the basic response, efficiency and power capacity relationships derived earlier are used below to develop system design procedures for two important cases: that of a fixed driver design, and that of only the final system specifications given.

Design with a Given Driver

One difficulty of trying to design an enclosure to "fit" a given driver is that the driver may be completely unsuitable in the first place. A convenient test of suitability for closed-box system drivers is provided by (51) and (54); the driver parameters must be known, or measured.

Equation (54) insists that the driver resonance frequency must always be lower than that of the system. If the designer wishes to avoid an enclosure which is wastefully large, i.e., he desires an air-suspension system, then α must be at least 3 and the driver resonance frequency must be no more than half the maximum tolerable system resonance frequency.

Similarly, Q_{TS} must be lower than the highest acceptable value of Q_{TCO}, and by approximately the same factor which relates f_S to the desired or highest acceptable value of f_C.

Finally, from (51), the value of V_{AS} must be at least several times larger than the enclosure size desired.

If the driver parameters appear satisfactory, the design of the system is carried out by selecting the most desirable combination of f_C and Q_{TCO} which satisfies (55) and then calculating α from (17). The required enclosure size (net internal volume) is then, from (51),

$$V_B = V_{AS}/\alpha, \quad (57)$$

or somewhat smaller if the enclosure is filled.

The reference efficiency is calculated from (23), and the acoustic power rating from (39) or (42). The electrical power rating is then, from section 5,

$$P_{ER} = P_{AR}/\eta_o. \quad (58)$$

Example of Design with a Given Driver

Using a standard baffle and unlined test enclosure, a European-made 12-inch woofer sold for air-suspension use is found to have the following small-signal parameters:

$$f_S = 19 \text{ Hz}$$
$$Q_{MS} = 3.7$$
$$Q_{ES} = 0.35$$
$$V_{AS} = 540 \text{ dm}^3 \text{ (19 ft}^3\text{)}.$$

Using (56) and (23),

$$Q_{TS} = 0.32$$
$$\eta_o = 1.02\%.$$

The manufacturer's power rating is 25 W, and the peak linear displacement is estimated to be 6 mm (¼ in). The effective diaphragm radius is estimated to be 0.12 m, giving $S_D = 4.5 \times 10^{-2}$ m^2 and $V_D = 2.7 \times 10^{-4}$ m^3 or 270 cm^3.

The values of f_S, Q_{TS} and V_{AS} for this driver appear to be quite favorable. The values of f_C, Q_{TCO} and f_3 to be expected from various suitable values of α are given in Table 1 together with the corresponding enclosure compliance V_{AB} (volume of an unfilled enclosure).

The $\alpha = 4$ alignment gives almost exactly a $B2$ response for an unfilled enclosure volume of 135 dm^3 or 4.8 ft^3. This would be quite suitable for a floor-standing system. The $\alpha = 9$ alignment gives excellent performance in a volume of only 60 dm^3 (2.1 ft^3). The $\alpha = 12$ alignment could probably be achieved in a 40 dm^3 (1.4 ft^3) enclosure with filling. Q_{TCO} would then be lower than shown, probably about unity, giving a cutoff frequency of about 53 Hz. This would be quite adequate "bookshelf" performance.

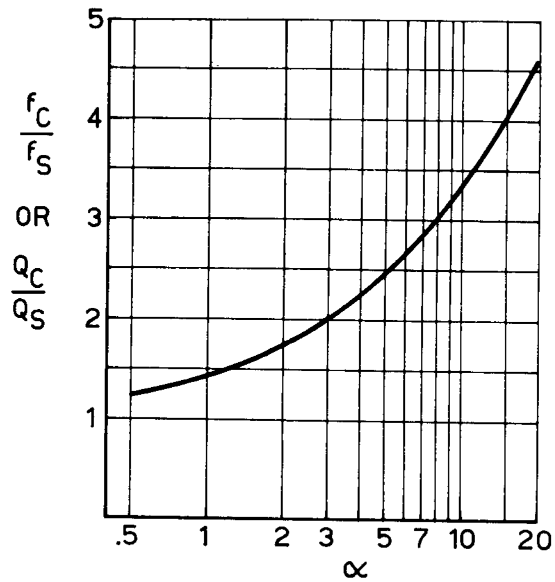

Fig. 13. Ratio of closed-box system resonance frequency and Q to driver resonance frequency and Q as a function of the system compliance ratio α.

Taking the larger $B2$-aligned system, the displacement-limited acoustic power rating for program material, from (42), is

$$P_{AR} = 0.19 \text{ W},$$

and the corresponding electrical power rating is

$$P_{ER} = 19 \text{ W}.$$

This is well within the power rating given by the manufacturer, so the system can safely be operated with an amplifier having a continuous power rating of 20 W.

The "bookshelf" design, because of its higher value of f_3, has displacement-limited ratings of about 0.5 W acoustical and 50 W electrical. This is much higher than the manufacturer's rating. In the absence of the actual value of $P_{E(max)}$ on which the manufacturer's rating is based, it is probably best to limit the amplifier power to 25 W. The system can then produce an acoustic output of 0.25 W.

Design from Specifications

Most engineering products are designed to suit specific requirements. Quite commonly, the "requirements" for a particular product contain conflicting factors, and the

Table 1. Expected Performance of the Given Driver

α	f_C, Hz	Q_{TCO}	f_3, Hz	V_{AB}, dm^3
4	42.5	0.72	42	135
6	50.3	0.85	44	90
9	60.0	1.01	47	60
12	68.6	1.15	50	45

engineer is called upon to assess the requirements and to adjust them to a condition of physical and economic realizability. Fig. 8, for example, frustrates the desires of many marketing managers who would be delighted to offer a one cubic foot (28 dm^3) air-suspension system giving flat response to 20 Hz at high efficiency.

The desired response of a closed-box loudspeaker system may be based on amplitude, phase, delay or transient considerations [13], but can always be reduced to a specification of f_C and Q_{TC}. Once the response is specified, either the enclosure volume V_B or the reference efficiency η_o may be specified independently; the other will then be determined or restricted to a minimum or maximum value. Finally, the power capacity may be specified in terms of either P_{ER} or P_{AR}. If both P_{ER} and P_{AR} must be fixed independently, this will detemine η_o and thus restrict V_B as above.

A typical set of design specifications might start with values of f_C, Q_{TC}, V_B and P_{AR}, together with a rating impedance which fixes R_E. Unless a special amplifier is to be used, it can be assumed that $Q_{TC} = Q_{TCO}$. Note that V_B effectively specifies the enclosure; the design problem is then to specify the driver.

The design process begins by assigning realistic values to Q_{MC} and α. The value of Q_{MC} has only a relatively minor effect on system performance through $k_{\eta(Q)}$. As noted in section 7, typical values are 2–5 for systems using filling material and 5–10 for unfilled systems. If no better guide to the expected value of Q_{MC} is available, assume $Q_{MC} = 5$. The required value of Q_{EC} for the system is then calculated from (9).

If maximum efficiency consistent with the initial specifications is desired, then the air-suspension principle must be used. This requires that α be at least 3 or 4, but its value will otherwise have only a small effect on system performance through $k_{\eta(C)}$ and may be chosen to have any value consistent with physical realizability of the driver. If α is chosen too large, the driver will be found to require unrealistically high compliance which, if realizable at all, may lead to poor mechanical stability of the suspension. A suitable choice of α is usually in the range of 3–10.

Next, the value of V_{AB} is established. This is equal to V_B for unfilled systems, but is increased by the factor $1.4/\gamma_B$ (typically 1.15 to 1.2) if the enclosure is filled.

The required driver small-signal parameters are then, from (17) and (18),

$$f_S = f_C/(\alpha+1)^{1/2}, \tag{59}$$

$$Q_{ES} = Q_{EC}/(\alpha+1)^{1/2}, \tag{60}$$

and

$$V_{AS} = \alpha V_{AB}. \tag{51}$$

V_{AT} is determined from (49). The reference efficiency to be expected from the completed system is calculated from (24). Alternatively, $k_{\eta(Q)}$, $k_{\eta(C)}$ and $k_{\eta(G)}$ may be evaluated separately and η_o determined from (26). The system electrical power rating P_{ER} is then calculated from (58). A comparable or lower value is assigned to $P_{E(max)}$, depending on the peak-to-average power ratio of the program material with which the system will be used.

The required value of V_D is calculated directly from (39) using Fig. 5 or (78) to determine $|X(j\omega)|_{max}$, or from (42), as appropriate. This value must be no larger than a few percent of V_B.

The driver is now specified by its most important parameters f_S, Q_{ES}, V_{AS}, V_D and $P_{E(max)}$ as well as its voice-coil resistance R_E which is typically 80% of the desired rating impedance. The system designer is faced with the problem of obtaining a driver which has the required parameters. If he has a driver factory available, he may have the required driver fabricated as described in the next section. If he does not possess this luxury, he must find a driver from among those available on the market.

At present, very few of the loudspeaker drivers offered for sale are provided with complete parameter information, either in the form above or any other. While this situation will no doubt improve with time, particularly as increasing demands are made on manufacturers to provide such information, today's system designer must obtain samples where possible and measure the parameters as described in [12]. The small-signal parameters should be measured with the driver mounted on a standard test baffle having an area of one or two square meters, e.g., [18, section 4.4.1], so that the diaphragm air load is approximately that which will apply to the driver in the system enclosure.

Example of System Design from Specifications

A closed-box air-suspension loudspeaker system to be used with a high-damping-factor amplifier is to be designed to meet the following specifications:

f_3	40 Hz
Response	$B2$
V_B	2 ft^3 (56.6 dm^3)
P_{AR}	0.25 W program peaks; expected peak/average ratio 5 dB.

The enclosure is to be lined, but not filled. It is assumed that the enclosure and driver losses will correspond to $Q_{MC} = 5$ and that it will be physically possible to obtain a compliance ratio of $\alpha = 5$.

The first two specifications translate directly into

$$f_C = 40 \text{ Hz}$$

and

$$Q_{TC} = Q_{TCO} = 0.707.$$

For $Q_{MC} = 5$, (9) gives

$$Q_{EC} = 0.824.$$

For $\alpha = 5$, $(\alpha+1)^{1/2} = \sqrt{6} = 2.45$, so from (59) and (60),

$$f_S = 16.3 \text{ Hz}$$

and

$$Q_{ES} = 0.336.$$

Also, for the unfilled enclosure, (51) gives

$$V_{AS} = 10 \text{ ft}^3 \text{ (283 dm}^3\text{)}.$$

Then, from (49),

$$V_{AT} = 1.67 \text{ ft}^3 \text{ (47.2 dm}^3\text{)}.$$

From (29), (30) and (31),

$$k_{\eta(Q)} = 0.858,$$
$$k_{\eta(C)} = 0.833,$$
$$k_{\eta(G)} = 1.36 \times 10^{-6}.$$

Thus

$$k_\eta = 0.97 \times 10^{-6}$$

and from (26),

$$\eta_o = 0.00351 \text{ or } 0.35\%.$$

The reference efficiency can also be calculated directly from (24) because f_C, V_{AT} and Q_{EC} are known.

The displacement-limited electrical power rating, from (58), is

$$P_{ER} = 71.5 \text{ W}.$$

An amplifier of this power rating must be used to obtain the specified acoustic output. For the expected peak/average power ratio, the thermal rating $P_{E(max)}$ of the driver must be at least 22.5 W.

Using (42) for the program power rating,

$$V_D = 3.4 \times 10^{-4} \text{ m}^3 \text{ or } 340 \text{ cm}^3.$$

This is only 0.6% of V_B, so linearity of the air compliance is no problem.

10. DRIVER DESIGN

General Method

The process of system design leads to specification of the required driver in terms of basic parameters. These parameters are used to carry out the physical design of the driver.

First, V_D must be divided into acceptable values of S_D and x_{max}. The choice of S_D may have to be a compromise among cost, distortion, and available mounting area.

The required mechanical compliance of the diaphragm suspension is then

$$C_{MS} = C_{AS}/S_D^2 = V_{AS}/(\rho_o c^2 S_D^2), \qquad (61)$$

and the required total mechanical moving mass is

$$M_{MS} = 1/[(2\pi f_S)^2 C_{MS}]. \qquad (62)$$

This total moving mass includes any mass added by filling material, as well as the air loads M_{M1} and M_{MB} on front and rear of the diaphragm. The latter can be evaluated from [1, pp. 216-217]. The mechanical mass of the diaphragm and voice-coil assembly is then

$$M_{MD} = M_{MS} - (M_{M1} + M_{MB}), \qquad (63)$$

less any allowance for mass added by filling material.

The magnet and voice coil must provide electromagnetic damping given by

$$B^2l^2/R_E = 2\pi f_S M_{MS}/Q_{ES}, \qquad (64)$$

or, for the value of R_E specified, a Bl product given by

$$Bl = (2\pi f_S R_E M_{MS}/Q_{ES})^{1/2}. \qquad (65)$$

This Bl product, together with the mechanical compliance, must be maintained with good linearity for a diaphragm displacement of $\pm x_{max}$. This effectively means that the voice-coil overhang outside the gap must be about x_{max} at each end. Also, the voice coil must be capable of dissipating as heat, without damage, an electrical input power $P_{E(max)}$. This design problem is familiar to driver manufacturers.

The driver parameter Q_{MS} usually plays a minor role in system performance, but it cannot be neglected entirely. The value of Q_{MS} in practical designs is often affected by decisions related to performance at higher frequencies. Where the driver diaphragm is required to be free of strong resonance modes at high frequencies, the outer edge suspension is usually designed to reflect a minimum of the vibrational energy travelling outward from the voice coil through the diaphragm material. This means that energy is dissipated in the suspension, and a low value of Q_{MS} results. The intended use of the driver or the constructional methods preferred by the manufacturer thus determines the approximate value of Q_{MS}. In a completed closed-box system, the value of Q_{MS} and the enclosure and filling material losses determine Q_{MC} and therefore the value of $k_{\eta(Q)}$ for the system.

Drivers for Air-Suspension Systems

It was stated earlier that the compliance ratio of an air-suspension system is not very important so long as it is greater than about 3 or 4. This means that the exact values of driver compliance, resonance frequency and Q are not of critical importance. It is in fact the moving mass M_{MS} and the electromagnetic damping B^2l^2/R_E that are of greatest importance. These can be calculated directly from the system parameters alone. Substituting (16), (17) and (18) into (61), (62) and (64), or using (3), (6), (8) and (25),

$$M_{MS} = S_D^2 M_{AC} = \rho_o c^2 S_D^2/(4\pi^2 f_C^2 V_{AT}), \qquad (66)$$

and

$$B^2l^2/R_E = 2\pi f_C M_{MS}/Q_{EC}. \qquad (67)$$

The exact value of mechanical compliance is not critically important so long as it is high enough to give approximately the desired compliance ratio. This is an advantage of the air-suspension design principle, because mechanical compliance is one of the more difficult driver parameters to control in production.

Example of Driver Design

The driver required for the example in the previous section has the following parameter specifications:

$$f_S = 16.3 \text{ Hz}$$
$$Q_{ES} = 0.336$$
$$V_{AS} = 283 \text{ dm}^3$$
$$V_D = 340 \text{ cm}^3$$
$$P_{E(max)} = 22.5 \text{ W}$$

The driver size will probably have to be at least 12 inches to meet the specifications of V_D and $P_{E(max)}$. This is checked by assuming a typical diaphragm radius of 0.12 m for the 12-inch driver, giving

$$S_D = 4.5 \times 10^{-2} \text{ m}^2.$$

For the required displacement volume of 340 cm³, the peak linear displacement must be

$$x_{max} = V_D/S_D = 7.5 \times 10^{-3} \text{ m} = 7.5 \text{ mm (0.3 in)}.$$

The total "throw" required is then 15 mm (0.6 in) which is realizable in a 12-inch driver. By comparison, the same displacement volume requires a throw of 22 mm (0.9 in) for a 10-inch driver, or 9.6 mm (0.38 in) for a 15-inch driver.

Continuing with the 12-inch design,

$$S_D{}^2 = 2.0 \times 10^{-3}\ \mathrm{m}^4.$$

The required mechanical compliance and mass are then, from (61) and (62),

$$C_{\mathrm{MS}} = 9.9 \times 10^{-4}\ \mathrm{m/N},$$
$$M_{\mathrm{MS}} = 97\ \mathrm{g}.$$

M_{MS} is the total moving mass including air loads. Assuming that the front air load is equivalent to that for an infinite baffle and that the driver diaphragm occupies one-third of the area of the front of the enclosure, the mass of the voice coil and diaphragm alone is

$$M_{\mathrm{MD}} = M_{\mathrm{MS}} - (3.14a^3 + 0.65\pi\rho_o a^3) = 87\ \mathrm{g}.$$

The magnetic damping must be, from (64),

$$B^2l^2/R_E = 30\ \mathrm{N \cdot s/m}\ \text{(MKS mechanical ohms)}.$$

For an "8Ω" rating impedance, R_E is typically about 6.5 Ω. The required Bl product for the driver is then

$$Bl = 14\ \mathrm{T \cdot m}$$

which must be maintained with good linearity over the voice-coil throw of 15 mm (0.6 in). The voice coil must also be able to dissipate 22.5 W nominal input power [12, eq. (6)] without damage.

Further examples of driver synthesis based on system small-signal requirements are contained in [28]; the method used is based on the same approach taken above but is arranged for automatic processing by time-shared digital computer. (The Thiele basic efficiency [17] used in this reference is based on a 4π sr free-field load and gives one-half the value of the reference efficiency used here.)

11. DESIGN VERIFICATION

The suitability of a prototype driver designed in accordance with the above methods may be checked by measuring the driver parameters as described in [12].[1] For an air-suspension driver, it is not necessary that f_S, Q_{ES}, and V_{AS} have exactly the specified values. What is important is that the quantities $f_S{}^2 V_{\mathrm{AS}}$ and f_S/Q_{ES}, which together indicate the effective moving mass and electromagnetic coupling, should check with the same combinations of the specified parameters. Then, if V_{AS} is large enough to give a satisfactory value of α for the system, the driver design is satisfactory.

Similarly, the completed system may be checked by measuring its parameters as described in section 6 and comparing these to the initial specifications.[1] The actual system performance may also be verified by measurement in an anechoic environment or by an indirect method [24].

12. CONCLUSION

The quantitative relationships presented in this paper make possible the low-frequency design of closed-box systems by direct synthesis from specifications and clearly show whether it is physically possible to realize a desired set of specifications. They should be useful to loudspeaker system designers who wish to obtain the best possible combination of small-signal and large-signal performance within the constraints imposed by a particular design problem.

These relationships should also be useful to driver manufacturers, because they indicate the range of basic driver parameters needed for modern closed-box system design and the extent to which costly magnetic material must be allocated to satisfy both the small-signal and large-signal requirements of the system.

Because the low-frequency performance of a completed system depends on a small number of easily-measured system parameters, it is always possible to specify—and verify—the low-frequency small-signal performance for standard free-field conditions. This information is of much greater value to users of loudspeakers than frequency limits quoted without decibel tolerances and without specification of the acoustic environment.

It is sincerely hoped that the quantitative relationships and physical limitations presented here—and in later papers for other types of direct-radiator systems—will not only be useful to system designers but will also contribute eventually to more uniform, realistic and accurate product specifications.

13. ACKNOWLEDGMENTS

This paper is part of the result of a program of postgraduate research into the low-frequency performance of direct-radiator electrodynamic loudspeaker systems. I am indebted to the School of Electrical Engineering of The University of Sydney for providing research facilities, supervision and assistance, and to the Australian Commonwealth Department of Education and Science for financial support.

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14. APPENDIX—SECOND-ORDER FILTER FUNCTIONS

General Expressions

Tables of filter functions normally give only the details of a low-pass prototype function. The corresponding high-pass or band-pass forms are obtained by suitable transformations. The general form of a prototype low-pass second-order filter function, $G_L(s)$, normalized to unity in the passband, is

$$G_L(s) = \frac{1}{s^2T_0{}^2 + a_1 sT_0 + 1}, \tag{68}$$

where T_0 is the nominal filter time constant, and the coefficient a_1 determines the actual filter characteristic. The corresponding high-pass filter function, $G_H(s)$, which

[1] A recent paper by Benson contains an improved method of Q measurement which compensates for errors introduced by large voice-coil inductance [32, Appendix 2]. The compensation is achieved by replacing f_C in eq. (45) of Part I of this paper— and f_S in [12, eq. (17)]—with the expression $\sqrt{f_1 f_2}$. The measured values of f_C and f_S are unchanged, and no other equations are affected.

preserves the same nominal time constant, is obtained by the transformation

$$G_H(sT_0) = G_L(1/sT_0). \qquad (69)$$

This gives the general high-pass expression

$$G_H(s) = \frac{s^2T_0^2}{s^2T_0^2 + a_1sT_0 + 1}. \qquad (70)$$

Equations (68) and (70) have exactly the same form as (20) and (19) for the displacement and response functions of the closed-box system. The two sets of equations are equivalent for

$$T_0 = T_C \text{ and } a_1 = 1/Q_{TC}. \qquad (71)$$

Study of the steady-state magnitude-*vs*-frequency behavior of filter functions for sinusoidal excitation is facilitated by using the magnitude-squared forms

$$|G_L(j\omega)|^2 = \frac{1}{\omega^4T_0^4 + A_1\omega^2T_0^2 + 1} \qquad (72)$$

and

$$|G_H(j\omega)|^2 = \frac{\omega^4T_0^4}{\omega^4T_0^4 + A_1\omega^2T_0^2 + 1}, \qquad (73)$$

where

$$A_1 = a_1^2 - 2. \qquad (74)$$

Cutoff Frequency

The half-power frequency $\omega_3 = 2\pi f_3$ of the high-pass function is obtained by setting (73) equal to ½ and solving for ω. Using (71) and (74), the normalized half-power frequency of the closed-box system is given by

$$f_3/f_C = \left[\frac{(1/Q_{TC}^2 - 2) + \sqrt{(1/Q_{TC}^2 - 2)^2 + 4}}{2}\right]^{1/2}. \qquad (75)$$

Frequencies of Maximum Amplitude

The frequency of maximum amplitude of either frequency response or diaphragm displacement is found by taking the derivative of (72) or (73) with respect to frequency and setting this equal to zero. This yields for the normalized frequency of maximum response

$$f_{G\max}/f_C = \frac{1}{[1 - 1/(2Q_{TC}^2)]^{1/2}} \qquad (76)$$

for $Q_{TC} > 1/\sqrt{2}$. For $Q_{TC} \leq 1/\sqrt{2}$, $f_{G\max}/f_C$ is infinite.

The normalized frequency of maximum diaphragm displacement is

$$f_{X\max}/f_C = [1 - 1/(2Q_{TC}^2)]^{1/2} \qquad (77)$$

for $Q_{TC} > 1/\sqrt{2}$. For $Q_{TC} \leq 1/\sqrt{2}$, $f_{X\max}/f_C$ is zero.

Amplitude Maxima

Substituting the above values of frequency into the expressions for $|G(j\omega)|^2$ and $|X(j\omega)|^2$ corresponding to (72) and (73), the amplitude maxima are found to be

$$|G(j\omega)|_{\max} = |X(j\omega)|_{\max} = \left[\frac{Q_{TC}^4}{Q_{TC}^2 - 0.25}\right]^{1/2} \qquad (78)$$

for $Q_{TC} > 1/\sqrt{2}$, and unity otherwise.

Types of Responses

The range of system alignments which may be obtained by varying Q_{TC} are thoroughly described in [13]. Particular alignments of interest, with brief characteristics, are:

Butterworth maximally-flat-amplitude response (*B*2) [13], [29]

$Q_{TC} = 1/\sqrt{2} = 0.707$, $f_3/f_C = 1.000$

Bessel maximally-flat-delay response (*BL*2) [13], [29], [30]

$Q_{TC} = 1/\sqrt{3} = 0.577$, $f_3/f_C = 1.272$

"Critically-damped" response [13]

$Q_{TC} = 0.500$, $f_3/f_C = 1.554$

Chebyshev equal-ripple response (*C*2) [13], [31]

$Q_{TC} > 1/\sqrt{2}$, other properties given by (75)-(78). A very popular alignment of this type is

$Q_{TC} = 1.000$, $f_3/f_C = 0.786$,
$|G(j\omega)|_{\max} = |X(j\omega)|_{\max} = 1.155$ or 1.25 dB.

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Note: Dr. Small's biography appeared in the December 1972 issue of the Journal.